ECON 3104 Exam 1

Wed. March 1, 2023

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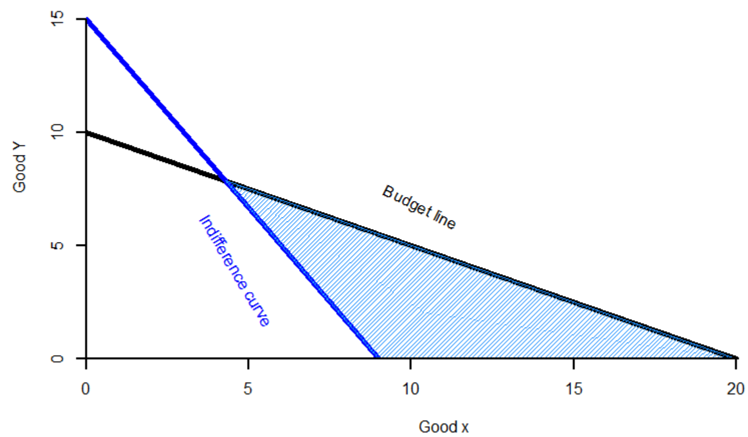
1. *(20pts.) Choice*: Consider a consumer with a utility function $U(x, y) = \sqrt{x} + \sqrt{y}$.
   (a) Find the quantity demanded for both goods if $p_x = 2,\ p_y = 4$, and $m = 40$
   - $w_x = \frac{40}{3},\ w_y = \frac{40}{12} = \frac{10}{3}$

2. *(40pts.) Perfect Substitutes*: A consumer has a utility function: $U(x, y) = 5x + 3y$
   (a) *10* Graph the indifference curve for the bundle of goods, B: $(x, y) = (6, 5)$
   (b) *10* On the same graph, plot the budget line for $p_x = 2,\ p_y = 4.$ and $m = 40$
   (c) *10* Shade in the area of the graph showing bundles the consumer both prefers over B and can afford for $p_x = 2,\ p_y = 4.$ and $m = 40$.



   (d) *10* Write the demand functions for both goods as functions of $p_x,\ p_y,$ and m.
   - With perfect substitute utility, consumers will spend all of their income on the good that gives the most utility per dollar so their demand is piece-wise depending on the price ratio. If the price ratio is exactly equal to the MRS, they are indifferent between any bundles along the budget line. [1] If your answer involved just two cases, $\frac{p_y}{p_x} < \frac{5}{3}$ and $\frac{p_y}{p_x} \geq \frac{5}{3}$, it is entirely sufficient.

$$w_x(m) \begin{cases} 0 & \frac{p_y}{p_x} < \frac{5}{3} \\ \text{Any} & \frac{p_y}{p_x} = \frac{5}{3} \\ \frac{m}{p_x} & \frac{p_y}{p_x} > \frac{5}{3} \end{cases} \quad ;\ w_y(m) \begin{cases} \frac{m}{p_y} & \frac{p_y}{p_x} < \frac{5}{3} \\ \text{Any} & \frac{p_y}{p_x} = \frac{5}{3} \\ 0 & \frac{p_y}{p_x} > \frac{5}{3} \end{cases}$$

3. *(20pts.) Slutsky Equation*: For perfect complements, $U(x, y) = \min(x, y)$, the Walrasian demand functions are $w_y(p_x, p_y, m) = w_x(p_x, p_y, m) = \frac{m}{p_x + p_y}$, and the Hicksian demand functions are $h_y(p_x, p_y, \bar{u}) = h_x(p_x, p_y, \bar{u}) = \bar{u}$

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[1] If we're splitting hairs, demand is actually a *correspondence*, not a function, meaning that it can give a set of possible choices as an output and not just a single value. The perfect answer specifies all of the feasible, utility maximizing bundles when $\frac{p_y}{p_x} = \frac{5}{3}$ but that is needlessly technical for this class.

(a) For these preferences, derive the Slutsky equation for the change in quantity demanded of good x with respect to a change in the price of x. Label the income effect and substitution effect portions.

$$\frac{\partial w_x(p_x, p_y, m)}{\partial p_x} = \frac{\partial h_x(p_x, p_y, \bar{u})}{\partial p_x} - \frac{\partial w_x(p_x, p_y, m)}{\partial m} w_x$$

$$\frac{m}{p_x + p_y}\frac{\partial}{\partial p_x} = (\bar{u}\frac{\partial}{\partial p_x}) - (\frac{m}{p_x + p_y}\frac{\partial}{\partial m})\frac{m}{p_x + p_y}$$

$$-\frac{m}{(p_x + p_y)^2} = 0 - \frac{1}{p_x + p_y}\frac{m}{p_x + p_y}$$

$$\underbrace{-\frac{m}{(p_x + p_y)^2}}_{\text{Total Effect}} = \underbrace{0}_{\text{Substitution Effect}} \underbrace{-\frac{m}{(p_x + p_y)^2}}_{\text{Income Effect}}$$

The Slutsky equation decomposition shows there is no substitution effect for perfect complements, which should match your intuition.

4. *(40pts.) Conceptual questions*

(a) *10* Explain why the two utility functions, $U_1(x, y) = ln(x) + ln(y)$ and $U_2(x, y) = xy$, represent identical preferences

- Utility is ordinal, not cardinal - unless we're working with expected utility problems. Any monotonic transformation represents the same preferences.

$$ln(U_2) = ln(xy) = ln(x) + ln(y) = U_1$$

$U_1 = ln(U_2)$ which is a monotonic transformation.

(b) *10* Suppose we want to use a Quasi-linear utility function to model a consumer's choice between slices of pizza and a composite good representing all other foods: $U(x, y) = ln(x) + y$. Which variable, x or y, should represent slices of pizza? Why?

- Pizza should be x and the composite good of 'all other food options' should be y. The single good would have diminishing marginal utility relative to the outside option.

(c) *10* Demand for cups of instant ramen, all else equal, is $w_r(p_r, m) = \frac{100}{p_r} - \frac{m}{5}$ for incomes $100 \le m \le 500$. Consider only incomes in this range.

- Is ramen a normal or inferior good? Explain.
  - Ramen is an inferior good. The derivative of demand with respect to price is always negative in this range of incomes. Consumption decreases as income rises, $\frac{\partial w_r}{\partial m} = -\frac{1}{5} < 0$
- Does ramen obey the Law of Demand? Explain.
  - Yes. The derivative of demand with respect to price is always negative: $\frac{\partial w_r}{\partial p_r} = -\frac{100}{p_r^2} < 0$

(d) *10* True or False: MRS is always equal to MRT at a consumer's optimal consumption bundle. If true, briefly explain why; if false, give an example.

- False. In fact, there are two counter examples in earlier questions. Both perfect compliments, (because optimal consumption is at the kink on the indifference curve,) and perfect substitutes, (because optimal consumption is usually all of one good or the other.) $MRS = MRT$ for interior solutions but, to be thorough, we have to check corner solutions as well.